STABILITY OF ECONOMIC SYSTEMS UNDER EXOGENOUS STRUCTURAL SHOCKS

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Motivation

- Fall of Lehman Brothers – “Too Connected to Fail”
- Networks and interaction among economic agents matter
  - Bottom-up approach
Motivation

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- Economics like analytically solvable models (often linear)
- Economics prefer global “equilibrium” approach with a representative agent (and further assumptions like market clearing)

However:

- Real life is full of non-linearity
- Representative agent is hard to define (man & woman – what is the representative agent?)
  - Mean-field theory
- There is no global equilibrium
  - Real economy is rather a complex self-organized system without global equilibrium
  - Interactions among economic agents matter
How to investigate possible non-linearity?

- Agent-based model – take it as an economy-like “laboratory”
  - Study the response of the model to the change in the underlying parameters of the model (technology change, taxation, exogenous policies)
  - Qualitative analysis of the response – not quantitative yet!

Problem: Agent-Based (Computational) Models lacks the analytic solution – simulating the “black box”

- Model of Employment, Production and Consumption
  - Leads to the empirically observed distributions (e.g. wealth in the society)
  - Uses two kinds of agents (citizens and endogenously generated companies)
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3 Thermodynamics of the EPC Model

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   - Structural variables
   - Economic variables

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3 THERMODYNAMICS OF THE EPC MODEL

4 NUMERICAL RESULTS
   • Structural variables
   • Economic variables

5 CONCLUSIONS
Model of Employment, Production and Consumption (EPC Model)

- The Agent-Based (Computational) Model modeling the basic economic activity in the economy
  - The citizens in our model work, earn salary and spend it on the market
  - The model mimics the flow of money among market, citizens and organizations through the perpendicular economic process
  - Process is composed of five consecutive steps – Commercial Cycle

The model was successful to get the distributions of the wealth in the society (workers vs. owners of the organizations)
Model II

Two types of agents:

- **Citizens:**
  - Represent people in the economy
  - Number is fixed

- **Organizations:**
  - Set up by citizens
  - Economic activity goes through them
  - Number dynamically varies
COMMERCIAL CYCLE

Perpendicular repetition of five consecutive steps:

- Hiring step
  - Matching between agents – employment/creating new companies
- Revenue step
  - Working for company – employees (and owner)
- Payment step
  - Paying wages and dividends
- Firing step
  - Reducing number of employees and going default
- Consumption step
  - Spending money on the market
Random Processes

We use “zero-intelligence” agents
- Decisions effectively described by appropriate random processes
- Independent, memory-less

**Type 1**: Probabilities for matching, getting revenue for organization and earning wage

\[ P(x) \propto \exp\left(-\frac{x}{T}\right) \]

with \( T \) being parameter – temperature.

**Type 2**: Consumption – uniform distribution.
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What do we understand under “different phases”? 

**Water:**
- Liquid
- Gas
- Solid state – ice

Changing the exogenous conditions – environment – we change the phases
- Change is non-linear
  - Reaching a “critical” temperature we change the phase discontinuously
Definition through free energy: The internal energy of a system less the energy that cannot be used to perform work.

Phase transition – discontinuity in the $n$-th derivative of the free energy with respect to some thermodynamic variable

- **First-order phase transition** – discontinuity in the first derivative
- **Second-order phase transition** – discontinuity in the second derivative
Critical point – phase transition ceases to exist

Source: http://serc.carleton.edu
We need free energy to study phase transitions in the model...
...or some variable defined as a first derivative of the free energy with respect to some thermodynamic variable

- **Entropy** satisfies that!
- We thus need a proper definition of the entropy – phases and phase transitions!

**Statistical thermodynamics** – statistical entropy

- Measures uncertainty (for the system to be spread over the different micro-states)

What are the micro-states which matter?
Monetary Entropy

Micro-states – distribution of money in the system!

Monetary entropy as

\[ S_{mon} = -k \sum_i P_i \log P_i, \]

where \( k \) is some constant, \( P_i \) is a probability of a micro-state \( i \).

- Micro-state as every individual agent in the economy – corresponding probability as her participation on the global wealth, i.e., \( P_i = M_i / M_{total} \), with \( M_i \) being amount of money possessed by agent \( i \).

Effective monetary entropy as

\[ S_{mon} = -k \left[ \frac{M_C}{M_{total}} \log \frac{M_C}{M_{total}} + \frac{M_X}{M_{total}} \log \frac{M_X}{M_{total}} + \frac{M_M}{M_{total}} \log \frac{M_M}{M_{total}} \right], \]

where \( M_C \) stands for the amount of money in the hands of citizens, \( M_X \) is the amount of money possessed by organizations and \( M_M \) is the amount of money resting on the market.
The money distribution in the society matters – different distribution of money thus defines phases:

- **Phase transition of the first kind:**
  - The entropy changes discontinuously as we vary the external parameters

- **Phase transition of the second kind:**
  - First derivative of the entropy changes discontinuously as we vary the external parameters
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Monte Carlo simulation techniques

Initialization:

- Number of citizens $N = 10000$.
- Every citizen starts with $W = 100$ units of money ($N \cdot W = 10^6$ units of money in the hands of citizens and the same amount on the market).
- Parameters – three temperatures, minimum wage, start-up money for new organization, share of wealth the organization keeps as reserves.

Simulation repeated $R = 100$ times, first $T = 2000$ steps for initialization.
We study the response of the system to change in three dimensions:

- **Minimum wage** (continuous variable)
  - Pressure of unions

- **Temperature of wages** (continuous variable)
  - Taxation, increase of the price of labor

- **Ownership structure** (structural change)
  - Owner works – direct ownership
  - Owner does not work – proxy ownership
    - Subsidy for companies of the size up to $l$ – to mimic the role of the owner by a constant subsidy (avg. contribution of an owner)
    - If $l = N$ – we replace random variable by its average (constant)
Results:

1 Structure of the economy
   1 Entropy
   2 Phase transitions & critical points

2 Economic implications
   1 Number and size of organizations
   2 Its market share
   3 Unemployment
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Note: Total entropy as a function of minimum wage and temperature of wage for four different economies, where \( l = -1 \) stands for economy with direct ownership, \( l = 10000, 1, 0 \) are proxy economies with subsidy to companies which has up to \( l \) employees.
Note: Phase diagram as a function of minimum wage and temperature of wage for four different economies, where $l = -1$ stands for economy with direct ownership, $l = 10000, 1, 0$ are proxy economies with subsidy to companies which has up to $l$ employees. Black line denotes a phase transition of the first kind while the phase transition without a black line denotes the phase transition of the second kind.
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Note: Relative number of organizations in the economy as a function of minimum wage and temperature of wage for four different economies, where $l = -1$ stands for economy with direct ownership, $l = 10000, 1, 0$ are proxy economies with subsidy to companies which has up to $l$ employees.
Note: Average number of employees per organization in the economy as a function of minimum wage and temperature of wage for four different economies, where $l = -1$ stands for economy with direct ownership, $l = 10000, 1, 0$ are proxy economies with subsidy to companies which has up to $l$ employees.
Revenue Captured by All Organizations Together

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Existence of the different phases – non-linearity in the economy-like systems!

- Non-linear response to the variation of the external variables
- Phase transitions:
  - First kind
  - Second kind
- Critical points

Question: Existence of one order parameter?
Conclusions

Critical Point & Policy

[Diagram showing a curve with points A and B connected by arrows, illustrating critical points and policy implications.]
CONCLUSION II

Broad consequences of the results:

- Corporate finance – change in the shareholders right:
  - The economy may react in a qualitatively different way
  - Maps of the phase transitions may completely change

- After-the-crisis policies:
  - Changing the entire economic system, however, it is hard to predict in what direction and whether it is really reducing the risk of abrupt non-linear changes
  - Any replacement of the dynamic behavior by either constant thresholds, cut-offs or subsidies – more instability of the system

- Findings strongly suggest that any reform changing part of the system may influence the entire system and completely change its response profile!
CONCLUSIONS

OUTLOOK

- Get the model closer to real economy
  - Fitting the model
- Our ultimate and (dream) goal – formulate (and solve) master equation
  - Analytic solution to the model