Simon, Bužek, and Gisin Reply: In his Comment [1] Bóna criticizes our derivation, in Ref. [2], of the basic rules of quantum dynamics, i.e., linearity and complete positivity, from what we called quantum statics and the condition that superluminal communication is forbidden. In particular, he claims that we implicitly use the projection postulate, and that our use of the no-signaling condition is redundant. Here we argue that both these claims are unjustified.

We would first like to clarify how quantum statics implies the possibility to prepare true *probabilistic mixtures of pure states* at a distance, even without the use of the projection postulate. Let Ψ_{AB} denote the initial pure entangled state whose spatially separated subsystems are locally controlled by Alice and Bob, respectively. Assume Bob performs a measurement represented, for simplicity, by an operator with a nondegenerated discrete spectrum. Assume he finds a result corresponding to the one-dimensional projector P_B. The projection postulate stipulates that immediately after the measurement, the state is represented by $\mathbb{1}_A \otimes P_B \Psi_{AB}$. We do not make any use of such an assumption. For example, our argument does also hold if Bob's measurement destroys his subsystem, as is the case in quantum optics experiments. Instead we argue as follows: Immediately after recording the result of his measurement, Bob knows the state in which Alice's subsystem is. This can be checked experimentally by challenging Bob to make the correct (statistical) predictions about all possible measurements that Alice may carry out on her subsystem (including the ones with deterministic outcomes). Now, if Bob knows Alice's state, then Nature must know it also. Admittedly, Alice does not know the state of her subsystem. But this is exactly like Alice not knowing who is behind her door when someone rings the bell.

A few seconds later, if Alice did not open the door, she still does not know who is behind her door. But she knows that it is the same person, merely a little bit older. Similarly, as long as Bob did not inform her, Alice still does not know the state of her subsystem. But she knows it is the same state as a few moments ago, merely a little bit older, i.e., evolved in time. Let us consider a fixed time interval $[t_0, t_1]$ and let us denote g the map that associates to any possible pure state P_ψ at time t_0 the evolved state $g(P_\psi)$ at time t_1. If Alice's information is such that at time t_0 she attributes nonzero probabilities p_j for several possible states P_{ψ_j} of her subsystem, then, at a later time t_1, if she did not receive any additional information in the meantime, she attributes the same nonzero probabilities p_j for the evolved states $g(P_{\psi_j})$. This is an immediate consequence of the fact that Alice is dealing with a probabilistic mixture. Let us emphasize that so far we have not learned anything about the form of the map g, for example it might be given by some nonlinear Schrödinger equation.

Our argument for the linearity of g in its action on pure states crucially depends on the no-signaling assumption. If Bob has a choice between two different measurements (with nondegenerate spectra, for simplicity), then, depending on his choice, Alice has one of the following mixtures: $\{p_j, P_{\psi_j}\}$ or $\{p'_j, P_{\psi'_j}\}$. Since they are derived from the same entangled state Ψ_{AB}, the two mixtures satisfy $\sum_j p_j P_{\psi_j} = \sum_j p'_j P_{\psi'_j}$. Now, if Alice merely waits for a short while, she holds one of following mixtures: $\{p_j, g(P_{\psi_j})\}$ or $\{p'_j, g(P_{\psi'_j})\}$. But, the no-signaling condition implies that, whatever the evolution her subsystem underwent, she should still not be able to distinguish between these evolved mixtures. As a consequence of our "quantum statics" assumption, the results of Alice's measurements at a given time will be completely determined by the density matrix of each mixture(in contrast to the time evolution, which for a general nonlinear theory will depend on the specific decomposition). Hence the density matrices after the time evolution have to satisfy

$$\sum_j p_j g(P_{\psi_j}) = \sum_j p'_j g(P_{\psi'_j}). \tag{1}$$

This has to hold for *all* probabilistic mixtures that Bob could have generated from Ψ_{AB}. It is not hard to show that this is possible only if g is a linear map on the pure states. Note that during the whole argument g can be thought of as defined exclusively for pure states. Of course, its extension to density matrices is immediate once its linearity has been established.

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Received 6 February 2003; published 21 May 2003
DOI: 10.1103/PhysRevLett.90.208902
PACS numbers: 03.65.Ta, 03.30.+p, 03.65.Ud

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