Comment on "No-Signaling Condition and Quantum Dynamics"

In a carefully worded Letter [1], the authors tried to derive linearity [i.e., affinity on "density matrices" (DM)] and complete positivity (CP) of general quantum mechanical dynamics $\mathbf{g}$ from usual (nonrelativistic) kinematics of quantum mechanics (QM), and from an additional "no-signaling condition" (NS). I shall try to show here that the declared goals of [1] were not attained there.

The authors consider a given system A in an arbitrary state described by a DM ρ_A, as a subsystem of a composed system AB occurring in a pure-state $|\psi_{AB}\rangle$. The subsystems A and B are spacelike separated. Different convex decompositions of the reduced DM $\rho_A = \sum_k q_k \rho_k$ are obtained by different choices of discrete measurements on B. "Measurements" of λI_B give trivial decomposition of ρ_A. The pure-state decompositions (corresponding to maximal measurements) are interpreted in [1] as representing the corresponding different "probabilistic mixtures" (PM) in the sense of (classical) statistical ensembles (of quantal systems), sometimes in literature called *Gemenge*, or also *genuine mixtures* in [2].

The evolution $\mathbf{g}$ ("not necessarily linear") of A "is *a priori* defined only on pure states...," $\mathbf{g}$: $P_\psi \mapsto \mathbf{g}(P_\psi)$. An explicit extension of $\mathbf{g}$ to all considered states of A, e.g., to all decompositions $\{\rho_k, q_k\}$ of ρ_A, is essential, however, for the forthcoming discussion: Effects of any deterministic (no collapse) semigroup of time transformations $\mathbf{g}$ are supposed to be uniquely determined in QM by its initial conditions [$\mathbf{g}(P_\psi)$ could be a density matrix of A]. The following observation will also support my criticism, (a) "...the results of measurement on A will be completely determined by the reduced DM of the system" [1].

Decisive for proving linearity of $\mathbf{g}$ is (b) "...*every* PM of pure states corresponding to the DM ρ_A can be prepared via appropriate measurements on B" (this is supported by calculations of probabilities at A conditioned by results of measurements on B); such a process is classified in [3] as the "reduction of the wave packet," i.e., a use of the projection postulate (having an *ontological meaning*), that is, however, strongly rejected in [1].

The linearity of $\mathbf{g}$ is then implied by the relation

$$\mathbf{g}(\rho_A) = \mathbf{g}(\{\rho_k, q_k\}) = \sum_j p_j \mathbf{g}(P_{\psi_j}), \qquad (1)$$

[$\mathbf{g}(\rho_A) := \rho'_A(\{P_{\psi_j}, p_j\})$ in [1]], if valid for arbitrary (or at least pure) decompositions $\rho_A = \sum_k q_k \rho_k = \sum_j p_j P_{\psi_j}$; Eq. (1) was deduced in [1] from (b), and from a use of NS.

My criticism is concentrated on two points, i.e., mainly, to (first) criticism of the way of the deduction of the restriction (1) imposed on $\mathbf{g}(\{\rho_k, q_k\})$, leading to linearity of $\mathbf{g}$, and to, less important, (second) criticism of the statement of the implication: *{linearity & positivity (of each time evolution)}* $\Rightarrow$ *{complete positivity of* $\mathbf{g}$*}*.

First.—The necessity of (1) in [1] is given by mere "statics" of [1], without NS, since kinematics [embracing all "state space points •" appearing as initial conditions for $\mathbf{g}$, and also its values $\mathbf{g}(\bullet)$] does not contain in [1] any means (i.e., corresponding observables) to ascertain locally a distinction between different kinds of interpretation of ρ_A, cf. (a); then the value of $\mathbf{g}(\rho_A)$ should be here the same for ρ_A considered as an indecomposable quantity describing a quantum state of each single system A in an ensemble of equally prepared couples AB, as well as for ρ_A representing a specific ensemble of subsystems A each of which is in one of the states ρ_k taken from the set composing the chosen decomposition $\{\rho_k, q_k\}$ of ρ_A.

A state space for A can be introduced, however (as it was partly done implicitly in [1]), *consisting of all probability measures on density matrices* (interpreted as corresponding PM's, and encompassing different decompositions of the same density matrix as different points) with observables distinguishing them; let us define then $\mathbf{g}(\{\rho_k, q_k\}) := \sum_k q_k \mathbf{g}(\rho_k)$ for the case of PM $\{\rho_k, q_k\}$, and let $\mathbf{g}(\rho)$ be "independently" given for any (not decomposed) density matrix ρ, cf. [2], Sec. 2.1-e.. Then the proof of linearity of $\mathbf{g}$ in [1] (with a use of NS) depends on the possibility of an empirical check of (b) (i.e., of the existence of *physical* differences between different decompositions of ρ_A at the instant of the measurements on B) without using the results of measurements on B. Its negative result (due to NS) does not imply (1): All the physically indistinguishable "at a distance prepared PM's" are described by ρ_A and all of them evolve to $\mathbf{g}(\rho_A) \equiv \mathbf{g}(\sum_k q_k \rho_k)$ [$\not\equiv \mathbf{g}(\{\rho_k, q_k\})$ for nonlinear $\mathbf{g}$].

Second.—Assuming linearity and positivity of each physical time evolution transformation $\mathbf{g}$, authors infer CP of $\mathbf{g}$ by applying these properties to extensions AB of the considered system A. Their arguments consist, however, of a rephrasing of the definition of CP and of its physical motivation published in [4].

I infer that the authors did not succeed in their effort to prove in [1] the effectiveness of the new quantum mechanical axiom called "no-signaling condition," and the declared aims of the Letter [1] were not achieved.

Pavel Bóna
Department of Theoretical Physics FMPI
Comenius University, 842 48 Bratislava, Slovakia

Received 28 December 2001; published 21 May 2003
DOI: 10.1103/PhysRevLett.90.208901
PACS numbers: 03.65.Ta, 03.65.Ud, 05.20.Gg

[1] Ch. Simon, V. Bužek, and N. Gisin, Phys. Rev. Lett. **87**, 170405 (2001).
[2] P. Bóna, Acta Phys. Slovaca **50**, 1 (2000); revised versions in math-ph/9909022; cf. also quant-ph/9910011.
[3] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[4] E. B. Davies, *Quantum Theory of Open Systems* (Academic Press, London, 1976), Sec. 9.2.

 0031-9007/03/90(20)/208901(1)$20.00